Pravdepodobnost

Variace(bez opak.,zalezi na poradi) $v_{(n,p)} = \frac{n!}{(n-p)!}$

Permutace(vsechny prvky,zalezi na p.) $P = n!$ (moznost rozsazeni 4 lidi->4!)

Kombinace $C_{(n,p)} = \binom{n}{p} = \frac{n!}{(n=p)!\,p!}$

Variace s opak. $V'_{(n,p)} = n^p$

Permutace s opak. $C'_{(n,p)} = \binom{n+p-1}{p}$

Binom. rozvoj $(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + ... = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Pr: pomer tahu *a:b*; celkem tahu *s*; pravdep. vytaz. $a\ldots P = \frac{a}{a+b}$; $b\ldots q = 1-p$

jednotl.pravdep. $P_k = \binom{s}{k} p^k * q^{s-k}$; odchylky $\Delta = \sqrt{\frac{p*q}{n}}$

Normalni rozdeleni vs. skutecnost

Prumerny uzaver $v_u = \frac{[|u|]}{n}$

Pravdepodobny uzaver-median u_{med} stejny počet vetsi i mensi nez dana hodnota

Smerodatna odchylka $\sigma_u = \sqrt{\frac{[u^2]}{n}}$

Smerodatna odchylka smerodatne odchylky $\sigma\sigma = \sigma * \sqrt{\frac{2}{n}}$

Skutecna relat. cetnost $r = \frac{R}{n}$**,** R…cetnost v jednotl. intervalech

Pr: relat. pravdep.

int	R	r	norm.velicina t	G(v tabulkach)	P(rozdil G)
-3			3/σ	G_1	
	2	2/60			P_1
-2,4			2,4/σ	G_2	
	2	2/60			P_2
-1,8			1,8/σ	G_3	

Hranice 1-σ okoli $\sigma = \sqrt{p*(1-p)/60}$; pricist a odecist k pravdep.

Stredni hodnota $E_{(x)} = \int_{-\infty}^{\infty} xf(x)dx = \sum_{i=1}^{n} p_i x_i$ **;** p_i=pravdep.hodnot; x_i=hodnoty nahod.veliciny (odhadnu pomoci prumeru nebo medianu)

Variance $V_{(x)} = E(x-E(x))^2 = p*q = \frac{\sigma^2}{n}$ (odhadnu pomoci smerodat. odchylky)

Normalni rozdeleni Gaussova krivka $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

Zákon hromadeni skutecnych chyb

$$\varepsilon_p = \frac{\partial P}{\partial a} * \varepsilon_a + \frac{\partial P}{\partial b} * \varepsilon_b \text{ (P=a*b)}$$

Zákon hromadeni strednich chyb

$$\overline{m}_y^{\,2} = \sum_{i=1}^{n} f_{xi}^{\,2} * \overline{m}_{xi}^{\,2}$$

$$m_p^{\,2} = \left(\frac{\partial P}{\partial b}\right)^2 * m_b^{\,2} + \left(\frac{\partial P}{\partial c}\right)^2 * m_c^{\,2} + \left(\frac{\partial P}{\partial \alpha}\right)^2 * m_\alpha^{\,2} \;\; (P = \frac{bc\sin\alpha}{2})$$

$$\frac{\partial P}{\partial b} = \frac{c\sin\alpha}{2};\; \frac{\partial P}{\partial c} = \frac{b\sin\alpha}{2};\; \frac{\partial P}{\partial \alpha} = \frac{bc\cos\alpha}{2}$$

$$\overline{m}_y^{\,2} = \sum_{i=1}^{n} f_{xi}^{\,2} * \overline{m}_{xi}^{\,2}$$

nahodny vektor $X = \begin{pmatrix} x_1 \\ \dots \\ x_n \end{pmatrix}$ **; stred. hodnota** $\mu = \begin{pmatrix} \mu_1 \\ \dots \\ \mu_n \end{pmatrix}$

kovarian. mat. $Q_x = E * \begin{pmatrix} (x_1-\mu_1)^2 & (x_1-\mu_1)(x_1-\mu_2) & \dots \\ \dots & \dots & \dots \\ \dots & \dots & (x_n-\mu_n)^2 \end{pmatrix}$;mimo diag.=kovariance

pro nezavisle hodnoty je kovariance 0

Zákon prenaseni kovarianci

$Q_y = AQ_xA^T$; $A = \begin{pmatrix} \frac{\partial x_1}{\partial l_1} & \frac{\partial x_2}{\partial l_1} & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \frac{\partial x_n}{\partial l_n} \end{pmatrix}$; <u>A=$x$ radku;l sloupcu</u>; $Q = \begin{pmatrix} m_{x1}^{\,2} & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & m_{xn}^{\,2} \end{pmatrix}$

$$X = \begin{pmatrix} r \\ \alpha \end{pmatrix};\; x = r_0\cos\alpha_0;\; y = r_0\sin\alpha_0;\; A = \begin{pmatrix} \cos\alpha_0 & -r_0\sin\alpha_0 \\ \sin\alpha_0 & r_0\cos\alpha_0 \end{pmatrix};\; Q_x = \begin{pmatrix} \sigma_r^{\,2} & 0 \\ 0 & \sigma_\alpha^{\,2} \end{pmatrix}$$

<u>pr:</u> $L = \begin{pmatrix} d_A \\ d_B \\ \alpha \\ \omega \end{pmatrix}$; $X = \begin{pmatrix} X_A \\ X_B \\ Y_A \\ Y_b \end{pmatrix}$; $x_A = d_A\cos\alpha$; $x_B = d_B\cos(\alpha+\omega)$; $y_A = d_A\sin\alpha$

$$y_B = d_B\cos(\alpha+\omega);\; A = \begin{pmatrix} -\cos\alpha & 0 & -d_A\sin\alpha & 0 \\ \sin\alpha & 0 & d_A\cos\alpha & 0 \\ 0 & \cos(\alpha+\omega) & -d_B\sin(\alpha+\omega) & -d_B\sin(\alpha+\omega) \\ 0 & \sin(\alpha+\omega) & d_B\cos(\alpha+\omega) & d_B\cos(\alpha+\omega) \end{pmatrix}$$

kov.matice mereni $Q = \begin{pmatrix} m_{dA}^{\ 2} & 0 & 0 & 0 \\ 0 & m_{dB}^{\ 2} & 0 & 0 \\ 0 & 0 & m_{\alpha}^{\ 2} & 0 \\ 0 & 0 & 0 & m_{\omega}^{\ 2} \end{pmatrix}$, na diag.dane stredni chyby

$Q_x = AQA^T$; stredni chyby souradnic jsou na diagonale Q_x

Vyrovnani

vektor mereni L; skutec.hodnoty mer. $L^{\wedge} = L + \varepsilon_e$, ε_e **=skutecna chyba; vyrovnane mereni** $\overline{L}$

vektor nezn. X; $X^{\wedge} = x(L) + \frac{\partial X}{\partial L} + \varepsilon_e = x + \varepsilon_x$; $C = \frac{\partial X}{\partial L} = \begin{pmatrix} \frac{\partial X_1}{\partial L_1} & \dots & \frac{\partial X_n}{\partial L_1} \\ \dots & \dots & \dots \\ \dots & \dots & \frac{\partial X_n}{\partial L_n} \end{pmatrix}$; $Q_x = CQ_l C^T$

linearizace $\overline{L} = L + v = L(x_0) + \frac{\partial l}{\partial x} * x$; $\frac{\partial l}{\partial x} * x = A$; derivace rce oprav podel x

($\overline{X} = X_0 + x$; X_0-pribl.sour.urc.b,x-opravy pribl.sour.)

$\sum |v_i|^2 p = v^T pv = \min$; $\frac{\partial v^t v}{\partial x} = 2A^T Ax - 2A^T l = 0 \Rightarrow AA^T x = A^T l$; $Q_l = AQ_x A^T$;

$Q_x = (A^T Q_l A\ )^{-1}$

linearz.rce oprav $v = Ax - l$; $l = L - L(X_0)$-reduk.mereni; $A = \frac{\partial l}{\partial x}$; $p_i = \frac{m_0^{\ 2}}{m_i^{\ 2}}$, m_0-volim,

m_i-znam; $P = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & \dots & 0 \\ 0 & 0 & p_n \end{pmatrix}$; $A^T PAx = A^T Pl$ ($A^T PA = (p_1 + \dots + p_n)$; $A^T Pl = p_1 l_1 + \dots + p_n l_n$)

$P = Q_l^{\ -1}$

<u>pr:</u>vzdalenost $X = (x)$; $L = \begin{pmatrix} l_1 \\ \dots \\ l_n \end{pmatrix}$; $x = l_i + v_i$; $v_i = x - l_i$ …linearni; $A = \begin{pmatrix} 1 \\ \dots \\ 1 \end{pmatrix}$; $n = A^T A$;

$A^T l = \sum_{i=1}^{n} d_i$; $\overline{d} = \frac{\sum d_i}{n}$

Vyrovnani mereni primych

vahy $p_i = \frac{m_0^{\ 2}}{m_i^{\ 2}}$ $l_i' = l_i - l_0$ …zavedeni redukce

vazeny prumer $\overline{x} = \frac{\sum p_i l_i}{\sum p_i}$ $d_x = \frac{\sum p_i l_i'}{\sum p_i}$; $x = l_0 + dx$; $v_i = d_x - l_i$'; $\sum p_i v_i = 0$

oprava $v_i = x_i - l_i$

jednotkova stredni chyba $m_0 = \sqrt{\frac{\sum p_i v_i^{\ 2}}{n-1}}$ **; str.ch.neznamych** $m_x = \frac{m_0}{\sqrt{\sum p_i}}$

*pr:*delka

$$X=(x);L=\begin{pmatrix}l_1\\ \dots\\ l_n\end{pmatrix};A=\begin{pmatrix}\frac{\partial x}{\partial l_1}\\ \dots\\ \frac{\partial x}{\partial l_n}\end{pmatrix}=\begin{pmatrix}1\\ \dots\\ 1\end{pmatrix};\Omega=\sum p_i v_i^{\,2}=\sum p_i(x-v_i)^2=\min;\bar{x};v_i;m_0;m_x;$$

kontrola $k=\sum Pv^T\cong 0;\sum p_i v_i^{\,2}=-\sum p_i v_i l_i;\sum p_i v_i^{\,2}=\sum p_i l_i^{\,2}-\frac{\left(\sum p_i l_i\right)^2}{\sum p_i}$

Dvojice mereni

stejne vahy

rozdil T a Z $d_i=l_i^{\,Z}-l_i^{\,T}$

prevyseni $p_i=\frac{l_i^{\,Z}+l_i^{\,T}}{2}$

vyrovnane vysky $H_i=H_{i-1}+p_i$

str.kil.ch.rozdilu $m_d=\sqrt{\frac{\sum d_i^{\,2}s_i}{n}}$

str.kil.ch.1mereni $m_0=\frac{m_d}{\sqrt{2}}$

str.kil.ch.vyr.prev $m_h=\frac{m_0}{\sqrt{2}}=\frac{m_d}{2}$

str.kil.ch.prev.konc.b. $m_x=m_0\sqrt{\sum s_i}$

str.ch.vysl.vyr.hodnot $m_{\bar{x}}=m_x\sqrt{n}$

ruzne vahy

$d_i^{\,\wedge}=d_i\sqrt{p_i};l_i^{\,\wedge Z}=l_i^{\,Z}\sqrt{p_i}$

$l_i^{\,\wedge T}=l_i^{\,T}\sqrt{p_i}$

$m_d=\sqrt{\frac{\sum d_i^{\,\wedge 2}s_i}{n}}=\sqrt{\frac{\sum p_i d_i^{\,2}s_i}{n}}$

$m_0=\frac{m_d}{\sqrt{2}}$

$m_h=\frac{m_0}{\sqrt{2}}=\frac{m_d}{2}$

$m_{\bar{x}}=m_x\sqrt{\sum q}$

Vyrovnani mereni zprostredkujicich

$\bar{L}_i=l_l+v_i$

$v_i=a_i dx_1+b_i dx_2+\dots+k_i dx_k+f_i\left(x_1^{\,0},\dots,x_k^{\,0}\right)-l_i,\ a_1=\frac{\partial f_i}{\partial x_1};dx_i=x-x^0,$

x^0=pribl.hodnota,x=promenna; $l_i'=f_i\left(x_1^{\,0},\dots,x_k^{\,0}\right)-l_i$ …redik.mereni

$v_i=Ax_i-l_i;\ v^T pv=\min$ (MNC); $\frac{\partial v^t Pv}{\partial x}=\left(\frac{\partial v}{\partial dx^T}\right)2Pv=A^T 2Pv=0$

$A^T PAx=A^T Pl$

$[paa]dx_1+[pab]dx_2+\dots+[pak]dx_k+[pal']=0$

$\dots$

$[pka]dx_1+[pkb]dx_2+\dots+[pkk]dx_k+[pl'l']=0$

Podminkove vyrovnani

podminek=nadbytecnych mereni

nezavislost podminek; maly počet promennych(->jednoduche podminky)

derivace podminek podle jednotl.mereni